Numerical correspondences between the physical constants

Christophe Réal

22, rue de Pontoise, 75005 PARIS

Abstract

We present here a note which synthesizes our previous ideas concerning some problems in cosmology, and the numerical correspondences between the constants in physics which we could deduce.

1 Tensorial quantum gravity

1.1 The equation

In [1], we consider Einstein's equations with a cosmological constant :

$$R_{ik} - \frac{1}{2}R + \Lambda g_{ik} = \kappa T_{ik} \tag{1.1}$$

We know that Λ and κ are constants, and the equation corresponds to the well known Hilbert-Einstein action with cosmological constant. We ask the question : is there some other equation, constructed from (1.1) by simple changes, thus still tensorial, which would take into account some quantum features, beyond classical gravity? Looking at the other interactions, we see that after renormalization, the coupling constants are running on energy (perturbative corrections) and that there also appears nonperturbative corrections, related to topology (instantons for example). Thus, we investigate (1.1) in the case in which κ is running on energy density, and for Λ being the Gauss-Bonnet topological term. More precisely we call $\tilde{\Sigma}$ the Gauss-Bonnet term and define $\tilde{\Sigma} = 4\tilde{\tilde{\Sigma}}$, which will simplify the calculations :

$$\tilde{\Sigma} = R^{(4)} - 4R^{(2)} + R^2 = 4\tilde{\tilde{\Sigma}} \tag{1.2}$$

where $R^{(4)} = R^{abcd}R_{abcd}$ and $R^{(2)} = R^{ab}R_{ab}$, R_{abcd} being the Riemann curvature tensor, R_{ab} the Ricci tensor, and R the scalar curvature. This term is on the left hand side of our equation and we obtain :

$$R_{ik} - \frac{1}{2}Rg_{ik} + \Lambda g_{ik} = \kappa(\epsilon)T_{ik} \tag{1.3}$$

with :

$$\Lambda = -\theta\tilde{\tilde{\Sigma}} + \Lambda_0 = -\frac{\theta\tilde{\Sigma}}{4} + \Lambda_0 \tag{1.4}$$

In a first case, we suppose θ constant, and we add a constant Λ_0. Then, in the general case, we suppose θ to be any function, for example $\theta = \theta(a)$, and we impose $\Lambda_0 = 0$. Equation (1.3) looks tensorial, classical, and nevertheless the new coupling of gravity has become dimensionless in one case. Indeed, using (1.3) and (1.4), we can prove that if the coupling $\kappa(\epsilon)$ takes the form :

$$\kappa(\epsilon) = \frac{\kappa_1}{\sqrt{\hbar c}\sqrt{\epsilon}} \tag{1.5}$$

then κ_1 a dimensionless real number. We will still note this constant κ_1 when considering $\hbar = c = 1$.

1.2 A remark

Equation (1.3) could appear inconsistent at first sight for the following reason. The left hand side of (1.1), associated to the constancy of κ, implies automatically the conservation of energy of the matter fields T_{ik}, which is not the case in equation (1.3). Thus, when considering equation (1.3), we also have to add to these equations the four equations of conservation of the matter fields. This does not make more equations than unknowns, because in (1.3) we add all matter fields as unknowns. In other words, (1.1) is the equation for the only gravitational field, and (1.3) is the equation for all fields. This explains why our equation gives the value of p, which has not to be put by hand anymore. Still, there could be a possible inconsistency in (1.3) : if (1.3) also governs the behavior of the matter fields, it could be in contradiction with the conservation of entropy. The present article answers to this question, since we prove here that in the case of the Robertson-Walker metric, (1.3) implies that the conservation of entropy is equivalent to the conservation of energy of the matter fields.

2 The predictions : constant theta

We emphasize that the equation (1.3) (associated with (1.4) and (1.5)), even if it does not come at first sight from any lagrangian, has very interesting properties concerning the description of the early universe. First, it possesses one equation more than the cosmological models based on general relativity. Indeed, we will see that this equation provides us with the relation

$$p = \epsilon/3 \tag{2.1}$$

which is not put by hand anymore but can be deduced from (1.3). As in the cosmological models based on general relativity, entropy is a consequence of the equations of movement. Above all, as far as the early universe is concerned (we are necessarily in the case $p = \epsilon/3$), the equation is a kind of unification of the standard cosmological model and of the inflationary universe. Indeed, we have

$$\epsilon \sim \frac{1}{a^4} \tag{2.2}$$

and $p > 0$, like in the standard model (these last two equations are consequences of (2.1) and of the conservation of entropy). But also, the behavior of $a(t)$ is like in the inflationary universe :

$$a(t) = \exp(Ht) \tag{2.3}$$

with H constant. So there is another consequence of (1.3) : it smoothes out the initial singularity with no need for a scalar particle. We emphasize that this way is the unique possibility to solve the problem of the beginning of the universe. The absence of a beginning cannot be due to a scalar particle belonging to the universe itself, because logically, the beginning is BEFORE the existence of the particle, which cannot then determine the absence of the beginning, by a kind of logical retroaction. We emphasize also that such an equation which gives p, obliges matter to be relativistic. In conclusion, we arrive in the case of constant θ to the conclusion : *one sole tensorial equation of quantum gravity governs at the same time the cosmological parameters of the universe, exactly as did the standard cosmological model, and the structure of the fundamental particles, giving the right relation between* p *and* ϵ*, whereas, furthermore, it smoothes out the initial singularity with no further hypothesis.*

3 The predictions : Varying theta (the general case)

3.1 The equations

In this case, the equation of quantum gravity is equivalent to a system of three equations, which are :

$$\frac{\ddot{a}}{a} = \frac{1}{4\theta(a)} \tag{3.1}$$

We also have, when (1.5) is used :

$$\frac{K}{a^2} + H^2 = \frac{2}{3}\kappa_1\sqrt{\epsilon} \tag{3.2}$$

and for the pressure :

$$p = \frac{\epsilon}{3}\left(1 - \frac{1}{\frac{2}{3}\kappa_1\sqrt{\epsilon}\theta(a)}\right) \tag{3.3}$$

3.2 An accelerating expansion

Equation (3.3) combined to the fact that we have on p the constraint :

$$0 \leq p \leq \frac{\epsilon}{3}$$

proves that necessarily :

$$\theta(a) > 0 \tag{3.4}$$

Using (3.1), we conclude that $\ddot{a} > 0$, which means that the expansion is accelerating. We emphasize that this experimental feature of our universe is not explained by the usual cosmological models in vigor, and that finding a model which could explain this fact has been the subject of intense researches since some time. This equation also solves the expansion problem since it explains why our universe is in expansion (because $\ddot{a}$ has ALWAYS been positive, so $\dot{a}$ has always been increasing, which explains why it has become positive).

3.3 The absence of a beginning

Our function a is convex, and of course positive, so we have two possibilities. With constant θ for example, we have an exponentially growing universe, with no beginning. In fact, we prove in [1] that we rather are, once taken into account ALL constraints on our model, in the situation with a universe shrinking to a minimum a_0, and expanding again from this value. We note $t = 0$ the cosmological time of the minimum.

3.4 The closed model

We necessarily are in the closed model, because at the minimum, or in any point where $\dot{a}$ is small enough, and since $K = \pm 1$, equation (3.2), which exhibits a positive quantity on the right hand side, implies $K = +1$ (closed model).

3.5 A new class of cosmological models

Equation (3.1) proves that any convex behavior of $a(t)$ can be reproduced by some positive function $\theta(a)$. Furthermore, we can change the behavior of $\kappa(\epsilon)$ by any other formula in which $1/\sqrt{\epsilon}$ of (1.5) is replaced by any function which has the dimension of a square of an energy. Another, even preferred choice would be :

$$\kappa(\epsilon) = \frac{\kappa_1^2}{H^2} \tag{3.5}$$

In all, we have two functions in our equation which can be chosen almost at will, in order to fit the whole set of cosmological experiments.

3.6 A particular choice of theta

We have studied all the models, only one, and even the choice (1.5) has not been our physically preferred choice, but the one which gave the simplest calculations! For a physical choice of $\theta(a)$, we refer to the results of constant θ, because in this case, $\sqrt{\theta}$, which is a length, is equal to $c/2H$. We thought that such a huge length could simply be the radius of the universe itself (which led us for the first time to suspect the fact that θ is in fact varying). Thus we suspected that $\sqrt{\theta} \approx a$, and we posed :

$$\theta(a) = \theta_0 a^2 \tag{3.6}$$

θ_0 being a constant, which will be proved in [3] to be around unity (with some interesting hypotheses, we can prove that $2\theta_0 = 1$.

3.7 Value of the time derivative of a

Using now (3.6), we can integrate (3.1) and find :

$$\dot{a}(t) = \frac{1}{\sqrt{2\theta_0}}\sqrt{\ln(a/a_0)} \tag{3.7}$$

Using for the relation $2\theta_0 = 1$, we can compute the maximal value of the present $\dot{a}(t_{pr})$, by taking for a_0 the Planck length, and we find : $\dot{a}(t_{pr}) \leq 11.7$. For the minimal value of

$a_{pr} = a(t_{pr})$, we can choose the condition $(a/a_0) \geq 6$, since it has been observed quasars with redshifts around $z = 5$. This leads to

$$1.34 \leq \dot{a}(t_{pr}) \leq 11.7$$

For a very probable value inside this interval, we choose for a_0 a value less than or equal to the radius of the early universe in the standard cosmological model ($z = 10^{10}$) :

$$\dot{a}(t_{pr}) \geq 4.8$$

What appears in any case is that our quantum cosmological model predicts for $\dot{a}$ a present value around unity.

3.8 The age of the universe

Observations in the context of the standard cosmology (Bennett, 2003, [5]) make appear the fact that the age of the universe should be very near the inverse of Hubble's constant. We note t the cosmological time, and it is a simple mathematical exercise to prove that from the behavior (3.7), we can prove, independently on the value θ_0, that

$$t \sim \frac{1}{H} \tag{3.8}$$

where we mean, by $f(t) \sim g(t)$, that the ratio of these two functions tends to 1 when $t \rightarrow +\infty$.

3.9 The cosmological constant problem

3.9.1 The classical context

As far as the standard model of cosmology is concerned, the cosmological parameters of the model are measured with a very good approximation (Bennett and al. 2003, [5]). In particular, there are in this model two important parameters, the total energy density $\Omega_{TOT} \approx 1$ and the energy density of dark energy Ω_Λ, the observed relation being :

$$\Omega_\Lambda \approx \frac{3}{4}\Omega \tag{3.9}$$

There is a lot of dark energy density, which remains unexplained. Furthermore, the model uses the equation of general relativity, with a cosmological constant Λ :

$$R_{ik} - \frac{1}{2}Rg_{ik} - \Lambda g_{ik} = 8\pi G T_{ik} \tag{3.10}$$

Now the term Ω_Λ is defined by the formula :

$$\Omega_\Lambda = \frac{\Lambda}{3H^2} \approx \frac{3}{4} \tag{3.11}$$

The cosmological constant problem is to understand why a constant like Λ should be nonzero, and furthermore should possess such a tiny strictly positive value :

$$\Lambda \approx \frac{9H^2}{4} \tag{3.12}$$

Finally, we can state the problem in the following way : the equations of general relativity are in the number of two, one which gives ϵ, the other gives p. We have, furthermore, the equation of conservation of entropy, so three equations plus the fact that they are dependent. So we choose two equations, say the conservation of entropy and :

$$R_0^0 - \frac{1}{2}R - \Lambda = 8\pi G\epsilon \tag{3.13}$$

If we pass the constant Λ to the right hand side of the equation, and insert it in the term in ϵ, we find a new ϵ, which we could call ϵ_{app}, because it is an apparent energy density. The observed value of Λ in the classical context is such that :

$$\epsilon_{app} = 4\epsilon \tag{3.14}$$

As a remark, ϵ_{app} is the value of the observed energy density, when the equation without the cosmological term is used, that is to say we have

$$R_0^0 - \frac{1}{2}R = 8\pi G\epsilon_{app}$$

The other equation just gives the relation between p and ϵ. If we want this conservation of entropy still to be valid for apparent quantities, we have to pose

$$p_{app} = 4p$$

Since in the case of the standard model we have $p = 0$, this does not change anything for the value of the pressure.

3.9.2 The quantum context

In the context of the quantum equation, we know the origin of the Λ term : it is the Gauss-Bonnet term which we have inserted in order to take into account some kind of nonperturbative (topological) quantum corrections to classical gravity. The computations made in [1] prove that we have, for $G_0^0 = R_0^0 - \frac{1}{2}R$:

$$G_0^0 = 3\left(\frac{K}{a^2} + H^2\right) \tag{3.15}$$

In the quantum context, we also have two equations, plus the conservation of entropy, and they also are dependent. We can choose the equation ([1]) :

$$G_0^0 + \Lambda = \frac{3}{2}\left(\frac{K}{a^2} + H^2\right) = \kappa_1\sqrt{\epsilon} \tag{3.16}$$

and the conservation of entropy. Then, we have to analyze (3.16), and how ϵ is affected by forgetting the Λ Gauss-Bonnet term. If we forget the Λ term in (3.16), we have to replace

$$G_0^0 + \Lambda = \kappa_1\sqrt{\epsilon}$$

by :

$$G_0^0 = \kappa_1\sqrt{\epsilon_{app}} \tag{3.17}$$

where ϵ_{app} is the apparent matter density, exactly as we did in our analysis of the case of general relativity. The difference is that now ϵ_{app} can be calculated theoretically from the quantum equations and compared to the original ϵ. In [1] we prove that :

$$\Lambda = -\frac{1}{2}G_0^0 \tag{3.18}$$

Forgetting Λ in our equation would have the net effect of changing

$$G_0^0 + \Lambda = \frac{1}{2}G_0^0$$

for G_0^0. So we see that the net effect of forgetting the Λ-term on the left hand side of the equation is to multiply the right hand side by 2, which has the effect of doubling κ_0, if we interpret this change in terms of a change of the gravitational constant. However, if we prefer to interpret the change in the equation as a change in ϵ, we get the striking relation :

$$\epsilon_{app} = 4\epsilon \tag{3.19}$$

This factor 4 corresponds to a prediction of the quantum equation in the quantum context, and is equal to the factor 4 coming from the observations, in the context of general relativity.

3.9.3 Complete calculation of the cosmological constant

If we compute Ω_Λ using the fact that our observations of the values of the masses in the cosmos are only based on the principle of inertia, that is to say, are based on what we see of the deformations of spacetime from a flat metric, we find :

$$\Omega_{TOT} = \frac{G_0^0}{3H^3} \tag{3.20}$$

We recall that we had (3.18) :

$$\Lambda = -\frac{1}{2}G_0^0 \tag{3.21}$$

and from (3.15) and (3.16) :

$$G_0^0 = 3\left(\frac{K}{a^2} + H^2\right) = 2\kappa_1\sqrt{\epsilon} \tag{3.22}$$

So we obtain :

$$\Omega_{TOT} = \frac{2\kappa_1\sqrt{\epsilon}}{3H^2} \tag{3.23}$$

We know that Λ is negative because it possesses an extra minus sign compared to the usual Λ of general relativity. Putting all these relations together, we find that our equation predicts for the usual Λ a positive value, verifying :

$$\Omega_\Lambda = \frac{\Lambda}{3H^2} = \frac{1}{2}\Omega_{TOT} \tag{3.24}$$

which is clearly in the domain of uncertainties of the observations, since this domain is determined by the relations

$$-1 < \frac{\Lambda}{3H^2} < 2$$

With $\Omega = 1.02$ (observations of Bennett, 2003, [5]), our Λ is just at the center of the former interval.

3.10 Determining kappa

3.10.1 The other kappa

We now observe that the coefficient 4 between the true physical and apparent energy densities is only 4 because it is viewed from the place of ϵ, under the square root. Of course this coefficient becomes 2, viewed from the place of κ_1, or even from the place of Λ, that is to say outside the square root. The interpretation of this factor 4 depends on how the quantum equation is established in the context of unification, and depends on the origin of the dependence of the gravitational coupling G on ϵ. Furthermore, the problem of these coefficients would not have appeared in other quantum cosmological models, the relation (3.5) does not make appear any square root for example. In fact, equation (1.5) renders difficult to compare general relativity and our quantum equation. To simply the problem, we just recall that the net effect of Λ is to multiply the right hand side by 2. This multiply κ_1 by 2 or ϵ by 4. In general relativity, multiplying the right hand side by 2 would have multiply ϵ by 2, and this behavior, inserted in the quantum (3.16), would have multiply κ_1 by $\sqrt{2}$. So, we have a incertitude on the value of κ_1 of a factor like $\sqrt{2}$ (or 2 because Λ also multiply κ_1 by 2). Maybe equation (3.5) can solve this problem, however simple relations below will show that the fundamental constant is κ_0, given by the relation :

$$\kappa_0 = 2\sqrt{2}\kappa_1 \tag{3.25}$$

3.10.2 Value of the kappas

The flatness problem is equivalent to prove that the total energy density of matter, $\Omega = \Omega_{TOT}$, that is to say the energy density when dark energy is taken into account, has a present value near unity. In classical gravity, the value of Ω_{TOT} takes the form

$$\Omega = \Omega_{TOT} = \frac{8\pi G\epsilon_{app}}{3H^2} \tag{3.26}$$

A reasonable relation between G and κ_1 is found by comparing general relativity and quantum gravity. In quantum gravity, we have :

$$\frac{K}{a^2} + H^2 = \frac{2}{3}\kappa_1\sqrt{\epsilon} \tag{3.27}$$

In general relativity the relation was :

$$\frac{K}{a^2} + H^2 = \frac{8\pi G}{3}\epsilon_{app} \tag{3.28}$$

We also have $\epsilon_{app} = 4\epsilon$, which is a consequence of the quantum equation of gravity. Inserting this relation in (3.28) (this relation is also valid in the classical case in virtue of the observations), we obtain :

$$\frac{K}{a^2} + H^2 = \frac{32\pi G}{3}\epsilon \tag{3.29}$$

Comparing (3.27) and (3.29) we find :

$$\kappa_1 = 16\pi G\sqrt{\epsilon} \tag{3.30}$$

Using (3.23), we find :

$$\kappa_1 = \frac{3\Omega H^2}{2\sqrt{\epsilon}} = 16\pi G\sqrt{\epsilon} \tag{3.31}$$

and κ_1 is also the square root of the two expressions in (3.31) which leads to :

$$\kappa_1 = \sqrt{24\pi}G^{1/2}(\Omega^{1/2}H) \tag{3.32}$$

Using finally (3.25) we obtain the value of κ_0 :

$$\kappa_0 = \sqrt{192\pi}G^{1/2}(\Omega^{1/2}H) \tag{3.33}$$

Or, we can deduce the value of Newton's constant :

$$G = \frac{1}{192\pi}\frac{\kappa_0^2}{(\Omega^{1/2}H)^2} \tag{3.34}$$

Using the observed values of Ω_{TOT} and H (Bennett, 2003, [5]), we find :

$$\kappa_1 = 1.087 \times 10^{-60} \tag{3.35}$$

and :

$$\kappa_0 = 3.074 \times 10^{-60} \tag{3.36}$$

3.11 The flatness problem

We have to prove that the expression for Ω_{TOT} tends to a finite value when $t \to +\infty$. We know that the present value of $\dot{a}$ is $\dot{a} = \lambda$. We thus find

$$\frac{1}{a^2} = \frac{H^2}{\lambda^2}$$

Finally, defining

$$\mu = \frac{\lambda^2}{\lambda^2 + 1} \tag{3.37}$$

we obtain :

$$\frac{1}{\mu}H^2 = \left(\frac{K}{\lambda^2} + 1\right) H^2 = \left(\frac{K}{\dot{a}^2} + 1\right) H^2 = \frac{K}{a^2} + H^2 = \frac{2}{3}\kappa_1\sqrt{\epsilon}$$

such that :

$$\frac{1}{\mu}H^2 = \frac{2}{3}\kappa_0\sqrt{\epsilon} \tag{3.38}$$

We now use the value of Ω_{TOT} in (3.23) (which also can be deduced from (3.26) and (3.30)), and find :

$$\Omega = \Omega_{TOT} = \frac{2\kappa_1\sqrt{\epsilon}}{3H^2} = \frac{1}{\mu} \geq 1 \tag{3.39}$$

Here we recall that we are necessarily in the closed model (section 3.4). We thus have :

$$\frac{1}{\mu} = \frac{1}{\lambda^2} + 1 \tag{3.40}$$

in such a way that $1/\mu \approx 1$ and $1/\mu \geq 1$. When λ is not used anymore to note the present value of $\dot{a}$, but rather its limit when $t \to +\infty$, our result is not the present value of Ω but its limit value. The present value have been observed to be, in the context of the standard cosmological model (Bennett and al., 2003) :

$$\Omega = 1.02 \pm 0.02 \tag{3.41}$$

To find $\Omega = 1.02$ in the quantum context, we need the present value of $\dot{a}$ to be

$$\dot{a}_{pr} = \lambda = 7.07$$

and to find the greatest possibility $\Omega = 1.04$ we need

$$\dot{a}_{pr} = \lambda = 5$$

A remark can be made : if it can be observed, in our universe, distances of the order of $200Mpc$, and if the cH^{-1} distance is about $4000Mpc$, we then are sure that $\dot{a} \geq 1/20 = 0.05$. That the universe could be one hundred times bigger than this minimum value does not seem a priori to be ruled out by any experiment, and only very small values of $\dot{a}$ are ruled out. We emphasize that no theory before the one presented here has been able to explain so precisely the value $\Omega = 1.02$ coming from observations.

4 Proof of a conjecture of De Broglie

4.1 Probing the structure of the particle using the conjecture

We suppose here De Broglie's famous hypothesis (De Broglie, 1963, 1963, [6]), that inside a fundamental particle, the temperature is equal, or at least proportional to the temperature. We further suppose that the total number of particles stays almost constant in the universe, and we prove that we necessarily are in one of our quantum models. If the former hypotheses are strictly verified, we even are in the $\theta \to +\infty$ model. Indeed, to prove this, we place ourselves in the case in which matter, we mean the set made of the usual fundamental particles we know, is non relativistic. We emphasize that these particles are the particles we know, and are supposed to be almost pointlike. Probing their structure means determining in they are made of relativistic stuff or not. They can be made for example of confined photons, in which case they still can be non relativistic in the sense that their global relative speeds are non relativistic, but inside, the value of the pressure still can be $p = \epsilon/3$. We see that the occupation number of this non relativistic gas, constituted of (almost) pointlike particles, is necessarily

$$N_m \sim \exp[-p^2/2mT_m] \tag{4.1}$$

where we take $k = 1$ for the Boltzmann constant, where T_m denotes the temperature of matter, and p are the particle momenta. As proved in Peebles, 1993, [7], the particle momenta are proportional to $1/a$, so we find that the occupation number is constant on the condition :

$$mT_m \sim \frac{1}{a^2} \tag{4.2}$$

We now probe the structure of particles, and use that the mass should be proportional to the temperature. With this further relation, $T_m \sim m$, We find from (4.2) :

$$m \sim T_m \sim \frac{1}{a} \tag{4.3}$$

This gives a behavior of the mass, analog to what we found from our quantum equation of gravity. We deduce even more, because this specific case in which $m \sim 1/a$, corresponds to $\epsilon \sim 1/a^4$, since the mass is equal to the energy for the non relativistic gas. Indeed, we have

$$\epsilon = \frac{N_m m}{V} \sim \frac{N_m m}{a^3} \sim \frac{1}{a^4}$$

where V is the total volume of the universe, because of (4.3), and also because N_m is constant. Using the conservation of entropy, this relation for ϵ means $p = \epsilon/3$, where now p is the pressure, and not anymore the momenta of the particles, as in (4.1). This last relation means that we are in the relativistic case. This does not contradict our first hypothesis, that the particles which we considered were non relativistic, but proves that their structure is relativistic. For example, we can suppose that these particles are made of more fundamental, relativistic and confined constituents, or simply made of photons. We also can leave this structure undefined, even if still relativistic, for the time being. From this relation $p = \epsilon/3$, and from relation (3.3) :

$$p = \frac{\epsilon}{3} - \frac{\sqrt{\epsilon}}{2\kappa_1 \theta} \tag{4.4}$$

proved in [1], we deduce that we are in the case $\theta \to +\infty$, which unifies all quantum models in a limit case.

4.1.1 Proof of De Broglie's conjecture

Inversely, starting from the hypothesis that $\theta \to +\infty$, using equation (4.4), we deduce that $p = \epsilon/3$, and thus that :

$$\epsilon \sim 1/a^4 \tag{4.5}$$

using the conservation of entropy. The particles we know are not observed to possess these relative relativistic velocities, which means that this relativistic behavior is not the consequence of the relative velocities of the usual fundamental particles, but rather a consequence of their structure. We thus can adopt the model which imposes that these known particles are made of more fundamental relativistic ones, the best candidate being of course the photon (we recall that the graviton will soon be expected to have no existence). Depending on the cosmological model, the relation $p = \epsilon/3$ is only approximate, or on the contrary, in the case $\theta \rightarrow +\infty$, rigorously verified. Since these baryons can be approximated by the picture of a non relativistic gas, we obtain (4.1), and the additional condition of conservation of their total number gives (4.2), where we still use [7]. Since the total mass $N_m m$ is proportional to the total energy ϵa^3, and since N_m is constant, we deduce from (4.5) the following relation, which is one part of (4.3) :

$$m \sim \frac{1}{a} \tag{4.6}$$

We now combine (4.2) and (4.6), and obtain $T_m \sim 1/a$. The last relation, compared to (4.6), gives :

$$T_m \sim m \tag{4.7}$$

Finally, the equation of quantum gravity gives a proof of the relation between the masses and the temperature, that is to say a proof of De Broglie's conjecture.

4.2 Conclusion concerning the masses

As proved in [1], the quantum equation of gravity governs at the same time the large scale of the universe and the structure of particles. We thus look for relations between the masses of the particles and the cosmological parameters. H has the dimension of an energy, a classical mass m have the dimension of an energy too, and we just saw that the associated quantum mass, noted $\tilde{m}$ from now on, is dimensionless. We thus look for a relation like :

$$m = \tilde{m} H \tag{4.8}$$

or like :

$$m = \tilde{m}(\Omega^{1/2}H) \tag{4.9}$$

where $\tilde{m}$ is the true constant of nature, in particular the true value of the mass, constant in respect to the cosmological time. The former relations are identical in the case $\Omega = 1$, and are very near from each other in the case $\Omega \approx 1$. Relation (4.9) corresponds to De Broglie's statement $m \sim T$. For the time being, we look at every possibility, and each of these possibilities corresponds to a different cosmological model, as we prove below. From (3.23), we deduce :

$$\Omega H^2 = \frac{2}{3}\kappa_1\sqrt{\epsilon} \approx \kappa_1\sqrt{\epsilon} \tag{4.10}$$

We also have

$$G \sim \frac{\kappa_1}{\sqrt{\epsilon}} \tag{4.11}$$

up to a constant 16π. So we see that

$$G^{1/2} \approx \kappa_1(\Omega^{1/2}H)^{-1} \tag{4.12}$$

The relations (4.9) and (4.12) permit us to retrieve that the gravitational charges are constant in time, taking into account that the true gravitational charges are the combinations $G^{1/2}m$ and not G or m. In this case, the model does not predict any observable variation of the gravitational charges. So this model does not predict any variation of the intensity of the gravitational interaction. In order to investigate all quantum models, the other formula (4.8) would, with this behavior of G, be a model with small variations of the intensity of the gravitational interaction. However, we still can use (4.8) and apply at the same time the principle of constancy of the intensity of the gravitational interaction. In this case, to obtain the constancy of the strength of the true gravitational charge $G^{1/2}m$, we have to suppose that G is strictly proportional to H^{-2}, and we find :

$$G \sim \frac{\kappa_1^2}{H^2} \tag{4.13}$$

At the end, we already have four different cosmological models, two choices for the behavior of G, and two choices for the behavior of m. The former relations shall be proved several times in this work, up to a term containing only $\dot{a}$, and we saw in [1] that in the quantum models, $\dot{a}$ has a present value around unity. We notice that, up to such a term, (4.11) and (4.13) (respectively (4.8) and (4.9)) are the same. In the limit case, when $\theta \to +\infty$, we find that $\dot{a}$ is constant, and all models tend to the same limit. This behavior of the masses, (4.8) or (4.9) or any other formula corresponding to (4.8) up to a factor containing only $\dot{a}$, corresponds to a solution to the mass gap problem of the Clay Mathematics Institute. If the masses are proportional to H, and if H tends to zero when $t \to +\infty$, we prove that the answer to the mass gap problem is no, that there can be no proof of a mass gap from the side of gauge theories, since there is no physical mass gap at all. On the contrary, if we try to apply the mass gap not anymore to m but to $\tilde{m}$, there is a mass gap, as we will prove, using only the Heisenberg's incertitude relations. Furthermore, the term containing only $\dot{a}$ up to which the formula is proved, will not change enough the behavior of the masses to be able to invalidate this proof.

5 A new constant of physics

5.1 Interpretation of Hubble's constant

In order to interpret the constant of gravitation, and later gravitation itself, we first need to interpret Hubble's constant, and this can be done remembering Heisenberg's uncertainty relations. From the relation

$$\Delta E \Delta t \geq 1 \tag{5.1}$$

we see that the uncertainty, in the observed value of any energy E, is linked to the interval of time of the observation Δt. But of course, this interval of time of the observation cannot be greater than the age of the existing universe itself. In the standard cosmological model, this value of the age of the universe is about H^{-1}. So we finally obtain :

$$\Delta E \geq \frac{1}{t} \approx H \tag{5.2}$$

Landau, in [8], explains that such an Heisenberg uncertainty relation proves that any strictly positive energy E cannot be smaller that H. Indeed, any energy below this value could not be distinguished from zero, because of the uncertainty. We conclude that H

is the smallest energy, strictly positive, ever possible in the universe. This, of course, corresponds to an energy gap, coming from cosmological necessities.

5.2 Interpretation of the constant of gravitation

We can give an interpretation of the constants κ_0 and κ_1 (we stick here to κ_0), analyzing (3.33) :

$$\kappa_0 \approx \sqrt{192\pi}\frac{H}{M_{Pcl}} \tag{5.3}$$

Here, $M_{Pcl} = G^{-1/2}$ is the classical Planck mass, and because $\Omega^{1/2} \approx 1$, we leave this term for the moment. To eliminate the term 192π, we note the quantum Planck mass :

$$M_{P0} = \frac{1}{\sqrt{192\pi}} M_{Pcl} \tag{5.4}$$

and we obtain :

$$\kappa_0 = \frac{H}{M_{P0}} \tag{5.5}$$

To analyze (5.5), we notice that M_{Pcl} has always been considered as an energy scale beyond which quantum gravity will come into play. $(M_{Pcl})^{-1}$, written as a distance, is believed to be the distance at which space-time breaks down. Now, in the former formula, there is H in the numerator, which is the smallest energy possible in the universe. On the other hand, κ_0, and its powers with exponents between -1 and 1 (see below), controls ratios of intensities of fundamental objects, for example fundamental interactions. The interpretation of the denominator is now clearer : this is the greatest energy possible of one particle, that is the physical cut-off in Feynman graphs which has been needed for so long, and κ_0 is the tiniest ratio possible, the smallest energy possible of one particle divided by the greatest energy possible of one particle. For this reason, H plays also the role of the infrared cut-off in Feynman integrals. κ_0 is the physical value corresponding to the breakdown that the mathematical real axis itself should have when it is applied to describe physics, and we believe that κ_0 should be used to try to explain the "quantum fact".

6 Electromagnetism and Gravitation

6.1 Introduction

In the large number hypothesis, we can find a relation between the mass of the pion and a cosmological parameter like Hubble's constant. At this stage, this relation could still be considered to be a mere coincidence, or on the contrary, to have a true physical meaning. We now prove that, not only this relation has a true physical meaning, but also that it belongs to a vast web of relations allowing us to compute all parameters of the standard model of particle physics. The relations of this web cannot be coincidences, first because of their great number, and second because of their astonishing accuracy.

6.2 Renormalization theory and tensorial quantum gravity

The principle of construction of the tensorial equation of gravity was that Newton's gravitational constant should depend on energy, to behave exactly like the couplings of the other interactions, after they have been renormalized. In the equation of quantum gravity, Newton's constant is proportional to :

$$G \sim \kappa(\epsilon) \sim \frac{\kappa_0}{\sqrt{\epsilon}} \tag{6.1}$$

The typical running of a coupling constant after renormalization, for example the running of α_{em} in Quantum Electrodynamics, is the following. The variations of α, between two energies μ_1 and μ_2, are described by :

$$\frac{1}{\alpha(\mu_1)} = \frac{1}{\alpha(\mu_2)} - \frac{2}{3\pi} \ln\left(\frac{\mu_1}{\mu_2}\right) \tag{6.2}$$

Clearly, it appears that the behaviors in (6.1) and in (6.2) are quite different. The first idea was to obtain a running of G analog to the running of α. We will see just below how this problem will find its solution.

6.3 The relation

The experimental value of the coupling of electromagnetism α_{em} is given by

$$\frac{1}{\alpha_{em}} = 137.036$$

with great accuracy. Here, α_{em}, which depends on energy, is observed at the energy of the mass of the electron. Now, using $\Omega = 1.02$ and $h = 0.71$ for the value of Hubble's constant (we recall h is defined by the formula $H = 100.h.km.s^{-1}.Mpc^{-1}$), we calculated the coupling constant κ_0 and we found in (3.35) and (3.36) : $\kappa_0 = 2\sqrt{2}\kappa_1 = 2\sqrt{2}\times 1.087 \times 10^{-60}$. Finally :

$$\kappa_0 = 3.074 \times 10^{-60} \tag{6.3}$$

To find what to do, we remember the interpretation we gave for κ_0. We obtained the relation

$$\kappa_0 = \frac{H}{M_{P0}}$$

We interpreted H as the smallest strictly positive energy that can carry one particle, and M_{P0}, the quantum Planck mass related to κ_0, as the greatest energy that can be carried by one particle, that is to say the true physical cut off in Feynman integrals. κ_0 was the dimensionless ratio representing the smallest physical ratio ever possible in our universe. We have found this way a solution to the problem of renormalization, since we have now a true physical cut off in Feynman integrals. Especially, in (6.2), there is no reason anymore to suppose that running α diverges when, for example, $\mu_1 \to +\infty$, simply because μ_1 can never be greater than the physical cut off. In other words, in (6.2), the ratio μ_1/μ_2 can never exceed the greatest ratio ever possible in the universe, which is, according to our interpretation, the ratio of the quantum Planck mass to the smallest energy defined by Hubble's constant, since H is the infrared cut-off in Feynman integrals. In other words, this greatest ratio is $(\kappa_0)^{-1}$. If our views are correct, the logarithm of this ratio should be of the order of the two other terms in equation (6.2), that is to say around 137. This way, we would have proved that our physical cut off has exactly the right order of magnitude to make quantum corrections of the numerical order of the quantities they correct. Furthermore, if the relation between $(\kappa_0)^{-1}$ and $1/\alpha_{em}$ is made via a logarithm, this explains perfectly the differences of behaviors in (6.1) and (6.2). This would explain why the running couplings possess logarithms in the quantum theory : the running of G on energy is a power law, this power law explains why we did not at first find a dimensionless Newton's constant, because this power law breaks the property of being dimensionless. Furthermore, taking the logarithm, this power law transforms into the usual running of the couplings after renormalization. At this stage, we take the logarithm of

$$\kappa_0 = 3.074 \times 10^{-60} \tag{6.4}$$

and find :

$$\ln\left(\kappa_0^{-1}\right) = 137.0321$$

This represents a relative uncertainty of only 2.8×10^{-5} from the observed value of α_{em}, so we have found the relation between the coupling constants of electromagnetism and gravitation :

$$\ln\left(\kappa_0^{-1}\right) = \frac{1}{\alpha_{em}} \tag{6.5}$$

This proves that the constant κ_0 does not only contain the data of the coupling constant α_{em}, but also the data of the mass of the electron, since at another energy, α_{em} takes a different value. This explains the large number hypothesis, because from the value $1/\alpha_{em} = 137.036$, we find :

$$\kappa_0 = \exp[-1/\alpha_{em}] \approx 3 \times 10^{-60} \tag{6.6}$$

In particular the term 10^{20} which appeared everywhere in this hypothesis was nothing else that $\exp[-1/3\alpha_{em}]$.

7 The mass of the electron

From the last relation, we can calculate the value of the constants of gravitation κ_0 and κ_1. We have of course

$$\kappa_0 = \exp\left[-\frac{1}{\alpha_{em}}\right] \tag{7.1}$$

and using the value of α_{em} given by CODATA [9] :

$$\frac{1}{\alpha_{em}} = 137.035999679(94) \tag{7.2}$$

we find

$$\kappa_0 = 3.06211514(29) \times 10^{-60} \tag{7.3}$$

Using $\kappa_1 = \kappa_0/2\sqrt{2}$, we obtain :

$$\kappa_1 = 1.08262119(11) \times 10^{-60}$$

We notice that from a relative uncertainty on the constant α_{em} which was 6.8×10^{-10}, the consequence of the exponential law is that the relative uncertainty on the gravitational coupling is 9.4×10^{-8}.

7.1 The value of Hubble's constant

From the former relation, and the particular model we used in [1], we are able to predict the value of Hubble's constant with great accuracy. More precisely, we can compute the expression $\Omega^{1/2}H$ to a relative precision of 1×10^{-4}, and H to a precision of 10^{-2}, while the relative precision given by Bennett and al., 2003, was 5×10^{-2}. We recall here that these formulas are available only in the particular quantum model we consider, and that many other models are possible, as we already showed. However, this kind of calculation shall permit us to determine which model we should choose, comparing the predictions with experiment. Here we use (3.33) $\kappa_0 = \sqrt{192\pi}(\Omega^{1/2}H)\sqrt{G}$ and find :

$$\Omega^{1/2}H = \frac{G^{-\frac{1}{2}}}{8\sqrt{3\pi}} \exp\left[-\frac{1}{\alpha_{em}}\right] = \frac{M_{Pcl}}{\sqrt{192\pi}} \exp\left[-\frac{1}{\alpha_{em}}\right] \tag{7.4}$$

From the list of the CODATA [9] recommended values of the fundamental constants, we obtain :

$$M_{Pcl} = 1.220892(61) \times 10^{19} GeV$$

We compute :

$$\Omega^{1/2}H = 1.522205(76) \times 10^{-42} GeV \tag{7.5}$$

To obtain the value of H, we recall that the value of Ω is between 1 and 1.04. We find for H :

$$H = 1.507(15) \times 10^{-42} GeV \tag{7.6}$$

We find the value of h :

$$h = 0.707(7) \tag{7.7}$$

which is again almost at the center of the interval of uncertainties of the cosmological observations (Bennett and al., 2003). We emphasize once again that these predictions only test one of our numerous quantum equations of gravity. Another problem is that the observed value of Ω may be model dependent. However, we stick to the observed interval of values since we showed in [1] that the quantum model gives theoretical predictions for the value of Ω which lie exactly in the same interval.

7.2 A formula for the mass of the pion

We know there exists an approximate relation for the mass of the pion :

$$m_\pi \approx \left(\frac{\bar{h}^2 H}{Gc} \right)^{\frac{1}{3}} \tag{7.8}$$

We now use now what we know, that is to say that the true mass of the pion is $\tilde{m}_\pi$, in the relation $m_\pi = \tilde{m}_\pi H$. We also use relation (3.34)

$$G = \frac{1}{192\pi} \frac{\kappa_0^2}{H^2}$$

(here we neglect the fact that Ω is not strictly equal to 1, which is an error less than only 2 percents). Replacing in (7.8), we obtain :

$$m_\pi = \left(\frac{H}{G} \right)^{1/3} \approx \left(\frac{(\Omega^{1/2} H)}{G} \right)^{1/3} = (192\pi)^{1/3} (\kappa_0)^{-\frac{2}{3}} (\Omega^{1/2} H) \approx (\kappa_0)^{-\frac{2}{3}} (\Omega^{1/2} H) \tag{7.9}$$

We recall that the term $\Omega^{1/2}$ can be present or absent in the equation, depending on the quantum model we choose. Thus, the simple relation which remains is :

$$\tilde{m}_\pi \approx (\kappa_0)^{-2/3} \tag{7.10}$$

We left in the last formula the term $(192\pi)^{1/3} \approx 8.45$ because first it is near unity, and second because it seems to be a consequence of the formula giving the classical Newton's constant G. So it appears that this term still belongs to (semi) classical physics. Without the numerical factor 192π, but with Ω, we find for the mass of the pion the value $7.22MeV$. This is in the domain of the masses of the known particles, but now much nearer to the mass of the electron. With the coefficient 192π we find the value $61MeV$, which is half the value of the mass of the pion. This confirms that the powers of κ_0 determine the fundamental quantities in the universe.

7.2.1 The tininess of gravitation

We now show that the former value for the masses explains the ratio of the gravitational to the electromagnetical forces. We call β_e and β_p the values of β corresponding respectively to the masses of the electron and the proton, in the formula :

$$\tilde{m} = (\kappa_0^{-1})^{\beta}$$

From the former section, we can say in first approximation that we have : $\beta_e = \beta_p = 2/3$. We compute :

$$\kappa_0^{2/3} = 2.1 \times 10^{-40}$$

We know from (3.34) that :

$$G = \frac{1}{192\pi} \frac{\kappa_0^2}{H^2}$$

and that

$$e^2 = 4\pi\alpha_{em}$$

So the ratio

$$\frac{Gm_p m_e}{e^2} = \frac{{\kappa_0}^2({\kappa_0}^{-1})^{\beta_p}({\kappa_0}^{-1})^{\beta_e}}{768\pi^2\alpha_{em}} \approx {\kappa_0}^{2/3} \qquad (7.11)$$

Indeed, we neglect the denominator, and we use $\beta_p = \beta_e = 2/3$. We arrive at :

$$\frac{Gm_p m_e}{e^2} \approx {\kappa_0}^{2/3} \approx 2 \times 10^{-40}$$

Inversely, we could have started from the last relation, and using (7.11) plus the condition $m_p \approx m_e$, that is to say $\beta_p = \beta_e$, we would have obtained :

$$2 - \beta_e - \beta_p = 2 - 2\beta_e = \frac{2}{3}$$

and thus :

$$m_e \approx m_p \approx {\kappa_0}^{-2/3} H \tag{7.12}$$

which is the right computation of the masses. Once again, a power of the constant κ_0 gives the ratio between the intensities of two interactions, and once again κ_0 appears as the principle of unification of the interactions. The formula for the masses and the formula for the ratio of the two interactions comes from only one principle, and are equivalent, as variations about different ways of writing 2/3 : $2 - 2/3 - 2/3 = 2/3$.

8 The mass of the electron

If we look carefully at our relation :

$$\frac{G m_e m_p}{\alpha_{em}} = {\kappa_0}^{\frac{2}{3}}$$

We note that

$$G = \frac{1}{192\pi} \frac{{\kappa_0}^2}{H^2}$$

and notice that $192\pi \approx 603$, which, up to a factor 3, is almost the famous ratio 1836 of the mass of the proton to the mass of the electron. Forgetting this factor 3, we write $m_p \approx 192\pi m_e$, and replacing in the equation we find

$$\left(\frac{m_e}{H}\right)^2 = \alpha_{em} {\kappa_0}^{-\frac{4}{3}} \tag{8.1}$$

which gives

$$m_e = \sqrt{\alpha_{em}} {\kappa_0}^{-\frac{2}{3}} H \tag{8.2}$$

We know that we can also write :

$$m_e = \sqrt{\alpha_{em}} {\kappa_0}^{-\frac{2}{3}} (\Omega^{1/2} H) \tag{8.3}$$

Using (7.2), (7.3), (7.5) and (8.3), we find $m = 0.617MeV$, which is quite near the observed mass of the electron. We have to remind ourselves that there can be also a coefficient $\sqrt{2}$ appearing in the formula. More precisely, if we look at the last relation, we find that it is the simplest relation possible for the mass of the electron. In the context of unification, the formula for the mass should be given only via the ratio m/e of the mass of the electron to its electromagnetic charge, and not directly via m alone. So the simplest formula, for the mass of the electron, is

$$\frac{\tilde{m}_e}{\sqrt{\alpha_{em}}} = (\kappa_0)^{-2/3} \tag{8.4}$$

where $\sqrt{\alpha_{em}}$ plays naturally the role of the charge of the electron. This is exactly formula (8.3), which gives a value just a little too high. However, the complete gravitational charge is clearly proportional to $\kappa_0 \tilde{m}_e = \sqrt{\alpha_{em}}(\kappa_0)^{1/3}$. In the quantum equation of gravity the effective coupling was κ_1, and not $\kappa_0 = 2\sqrt{2}\kappa_1$, so the true gravitational charge should not be $\sqrt{\alpha_{em}}(\kappa_0)^{1/3}$ but instead $\sqrt{\alpha_{em}}(\kappa_1)^{1/3}$, which is :

$$\frac{\sqrt{\alpha_{em}}}{\sqrt{2}}(\kappa_0)^{1/3}$$

Thus, the true formula for the mass of the electron is :

$$m_e = \sqrt{\frac{\alpha_{em}}{2}} \kappa_0{}^{-\frac{2}{3}} (\Omega^{1/2} H) \tag{8.5}$$

and this way, we find the value $m_e = 0.4360401MeV$. We add to this value the electron self-energy mass shift in second-order QED perturbation theory (Mandl, Shaw, [10]). We find :

$$\delta m = \frac{3e^2 m}{8\pi^2} \ln\left(\frac{\Lambda}{m_e}\right)$$

We take the mass Λ to be determined by $\beta_\Lambda = 1$, which is the additional hypothesis we made about the physical cut-off in Feynman integrals. We find then

$$\delta m = \frac{3m}{2\pi}(\beta_\Lambda - \beta_e) \tag{8.6}$$

Indeed, the classical formula is

$$\delta m = \frac{3m}{2\pi} \alpha_{em} \ln \frac{\Lambda}{m_e} \tag{8.7}$$

and

$$\Lambda = (\kappa_0^{-1})^{\beta_\Lambda}(\Omega^{1/2}H)$$

also

$$m_e = (\kappa_0^{-1})^{\beta_e}(\Omega^{1/2}H)$$

and

$$\alpha_{em}\ln[(\kappa_0)^{-1}] = 1$$

from the formula of unification. To compute the value of β_e, from $m_e = ({\kappa_0}^{-1})^{\beta_e}(\Omega^{1/2}H)$ we have to notice that the value of m_e that has really been computed is $0.436MeV$ and not the observed $0.511MeV$. So we have to use this 0.436 in the former formula to obtain the true value of β_e. This way, the new renormalized mass of the electron is

$$m_e = 0.5097MeV \tag{8.8}$$

which, compared to the observed

$$m_e = 0.511MeV \tag{8.9}$$

is inside the interval of uncertainties, taking into consideration that we have left aside corrections to higher orders. The relative uncertainty is 2.6×10^{-3}, which is very good.

9 Weak, Strong and Gravity

9.1 The Fermi constant

The fermi constant G_F has a simple expression in terms of κ_0. First we know that Newton's constant G reads :

$$G = \frac{1}{192\pi}\frac{\kappa_0^2}{(\Omega H^2)} \tag{9.1}$$

Second we know, from the CODATA [9] recommended values, that :

$$G_F = 1.16637(1) \times 10^{-5}(GeV)^{-2} \tag{9.2}$$

To obtain a dimensionless Fermi constant, which we still note G_F, we have to multiply this expression by $\Omega H^2 = (1.522205(76) \times 10^{-42} GeV)^2$. The dimensionless Fermi constant is equal to :

$$G_F = 2.70260533 \times 10^{-89} \tag{9.3}$$

In this value, appears clearly $(\kappa_0)^{3/2}$, and we find :

$$G_F = 5.04371(50)(\kappa_0)^{3/2} = \tilde{G}_F(\kappa_0)^{3/2} \tag{9.4}$$

Once again, up to a factor of order unity, one power of κ_0 determines another fundamental constant of nature. κ_0 governs the intensity of electromagnetism by its logarithm, giving α_{em}, it governed the strong interactions via the mass of the pion by $(\kappa_0)^{-2/3}$, and now governs the Fermi constant via $\kappa_0^{3/2}$. The value $\tilde{G}_F = 5.04371$ should be fundamental, and expressible in a simple manner. Also, when the dimensionless Fermi constant is not written with κ_0 but with κ_1, we find :

$$\begin{aligned} G_F &= 3 \times 0.99967(10) \times (\sqrt{2}\kappa_0)^{3/2} \\ &= 24 \times 0.99967(10) \times (\kappa_1)^{3/2} \end{aligned} \tag{9.5}$$

It is astonishing to find in this expression a number so near unity, even if the value 1 is ruled out by the precision of experiments. However, it appears that the powers of κ_0 clearly govern all values of the standard model of particles physics, which are now calculable up to a constant near unity, and that we need another theory at this stage to compute this constant. The same thing is going to happen in the calculation of the masses of the W and Z particles.

9.2 The masses of the W and Z particles

The theory of weak interactions exhibits the formula :

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g_W}{m_W}\right)^2 \tag{9.6}$$

where g_W is the coupling constant of the weak interactions, and m_W is the mass of the W particle. So we obtain :

$$g_W = 1.58803(5) m_W (\kappa_0)^{3/4} \tag{9.7}$$

From (9.6), the W particle verifies the relation :

$$m_W = g_W \left(\frac{G_F}{\sqrt{2}}\right)^{-\frac{1}{2}} \tag{9.8}$$

and since the only power of κ_0 appearing in the last formula is $(\kappa_0)^{3/2}$, coming from G_F, we find that m_W is governed by the power : $(\kappa_0)^{-3/4}$. So is m_Z, since the two particles have about the same mass. The simplest formula, that should give m_W, as the power $\kappa_0^{-3/4}$, is :

$$m_W = \frac{\sqrt{\alpha_{em}}}{2^{3/8}}(\kappa_0)^{-3/4}(\Omega^{1/2}H) \tag{9.9}$$

The complete gravitational charge is $\kappa_0 m_W \sim \kappa_0^{1/4} = 2^{3/8}\kappa_1^{1/4}$, which explains the factor $2^{3/8}$ in the denominator. In this formula, the term α_{em} is linked to the electric charge of the W particle, and we have to take its value at the energy m_W. The theory of unification of electro-weak interactions gives us relations between the masses of the W and the Z particles. In Pich, 1995, [11], the mass shifts of these two particles are computed. As far as the most important QED and QCD corrections to the mass are concerned, these corrections affect the relation between m_W and m_Z, but we do not know if in a more unified theory these changes would affect more specifically m_W or m_Z. In [11], m_Z, especially, is taken from experiment and kept constant all along the calculation. We do the same here, in other words m_Z, is not affected by these quantum corrections. The observed values of the two masses are $m_W = 80.403 GeV$ and $m_Z = 91.1887 GeV$ (Yao and al. 2006, [9]). We apply formula (9.9) to m_Z, an allow for a coefficient λ_Z, yet unknown and to be determined. We obtain :

$$m_Z = \lambda_Z \frac{\sqrt{\alpha_{em}}}{2^{3/8}}(\kappa_0)^{-3/4}(\Omega^{1/2}H) \tag{9.10}$$

To compute this theoretical value of the mass, we have to take the value of α_{em} at the energy m_Z. At m_Z, we have the value $(\alpha_{em})^{-1} = 128.8$ (Consoli, Jegerlehner, Hollik, 1989, [12]), and we find, using at first $\lambda_Z = 1$, the value :

$$m_Z = 44.68 GeV \tag{9.11}$$

Clearly, compared to the observed $m_Z = 91.1887 GeV$, we find that the coefficient $\lambda_Z = 2.057$, which is equal to 2 with a relative uncertainty of 2.9×10^{-2}, itself of the order of

$(\Omega - 1)$. We thus see that our simplest formula gives us unexpected good results, and seems to be able to probe the structure of the Z particle, predicting that this neutral particle is made of two particles, probably of respectively positive and negative electric charges.

9.3 Strong interactions

We know that the strong interactions are asymptotically free and that the strong coupling has the following expression :

$$\alpha_s(\mu) = \frac{1}{4\pi\beta_0 \ln(\mu^2/\Lambda_{QCD}^2)} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln(\ln(\mu^2/\Lambda_{QCD}^2))}{\ln(\mu^2/\Lambda_{QCD}^2)} \right] \tag{9.12}$$

(see for example [13], and see references therein). So, if we can compute the scale parameter Λ_{QCD} with the help of κ_0, the former formula unifies the coupling constant of strong interactions and gravity. The value of Λ_{QCD} is determined by experiments, once specified the subtraction scheme and the number of active flavors. For example $\Lambda_{\overline{MS}}^{N_f=5} = 217 \pm 24 MeV$. If we compare this mass to the power $\kappa_0^{-2/3}$, we find :

$$\Lambda_{\overline{MS}}^{N_f=5} \approx 30(\kappa_0)^{-2/3}(\Omega^{1/2}H) \tag{9.13}$$

It appears clearly that κ_0 by itself governs the order of magnitude of all parameters of the standard model of particles, and that these parameters should be completely calculated using only κ_0.

10 When GUT unify all four interactions

10.1 De Broglie's dimensionless spacetime

10.1.1 Unifying all scales in the universe

Equation (3.34)

$$G = \frac{1}{192\pi} \frac{\kappa_0^2}{\Omega H^2} \tag{10.1}$$

is exactly the type of formulas which unifies the smallest scales and the largest scales of the universe. Using (3.34) with the principle of constancy of the intensity of the gravitational interaction, we arrive at the conclusion that the masses are proportional to

$$m \sim \Omega^{1/2} H \tag{10.2}$$

which opens the possibility of the complete calculation of the masses of the fundamental particles using cosmological parameters. There is an important remark to make here : formula (10.1) will be proved completely in [3] using the following hypothesis : the calculations necessary to retrieve the abundances of the elements in the early universe are the same in our quantum model and in the standard cosmological model. Thus, the observations of the abundances of the elements in the universe, turn to be an experimental test of theories of unification. Nevertheless, to compare the standard and quantum cosmological models, we will have to REDEFINE SPACE AND TIME, and especially the running of time. To achieve this goal, we will stick essentially to De Broglie's analysis of a mass of a particle (and how he used this mass as a clock). For this reason, we have called this new spacetime De Broglie's dimensionless spacetime. Indeed, this space and time are completely dimensionless.

10.1.2 Unifying interactions

We now turn to the interpretation of formula (6.6) :

$$\kappa_0 = \exp\left[-\frac{1}{\alpha_{em}}\right] \tag{10.3}$$

Does there exist in physics such kind of law, and what can we deduce from this? Clearly this law is instanton-like, or also tunnelling-like. If we note g the coupling constant of a non-abelian Yang-Mills gauge theory, a self-dual solution to the euclidian Yang-Mills theory, an instanton, has in the path integral the contribution :

$$e^{-S} = e^{-8\pi^2/g^2} = e^{-2\pi/\alpha} \tag{10.4}$$

where $\alpha = g^2/4\pi$ (the instanton is only one possibility, we could say as well that we should try to show that gravity is a quantum tunnelling effect of electromagnetism). The behavior in (10.4) is analog to the behavior of the gravitational coupling in equation (10.3). We deduce that gravity is a nonperturbative relic of electromagnetism, and eventually of the other interactions. Thus, the most probable hypothesis is that there is no graviton in the universe.

10.1.3 Three new masses

We saw that when H is used as a unit of energy, there are two fundamental masses in the universe, the mass of the electron, which is governed by $(\kappa_0)^{-2/3}$ and the mass of the Z particle, which is governed by $(\kappa_0)^{-3/4}$. Thus there should be another mass

$$m = \frac{\sqrt{\alpha_{em}}}{2^{3/4}}(\kappa_0)^{-1/2}(\Omega^{1/2}H) \tag{10.5}$$

Also, there should be the mass :

$$m = \frac{\sqrt{\alpha_{em}}}{2^{3/2}}(\kappa_0)^{-1/3}(\Omega^{1/2}H) \tag{10.6}$$

Eventually, we can also construct a mass with $\kappa_0^{-1/4}$. The question is : is one of these masses of physical interest? The first mass is $m = \sqrt{\alpha_{em}} \times 6K$ (K for Kelvin) and seems fine to be the mass of the neutrino ν_e. The question is to know if the second, or the third mass, is, or is not, the mass of the photon.

11 The link with the Grand Unified Theories

11.0.4 Another formula

The gravitational charge of the electron is, using (8.4) :

$$\kappa_0 \tilde{m}_e \approx \sqrt{\alpha_{em}} \kappa_0^{1/3} \tag{11.1}$$

We recall the relation $m = \tilde{m}(\Omega^{1/2}H)$. We have :

$$\kappa_0 = \exp\left[-1/\alpha_{em}\right] \tag{11.2}$$

We know that $1/\alpha_{em} = 137.036$ and it appears that $1/\alpha_{GUT} \approx 45$ (see for example Ross, 1984, chapter 6, Fig 6.1, [14]). These numerical values lead directly to

$$\frac{1}{\alpha_{GUT}} \approx \frac{1}{3\alpha_{em}} \tag{11.3}$$

We thus deduce that the gravitational charge of the electron is

$$\frac{\kappa_0 \tilde{m}_e}{\sqrt{\alpha_{em}}} \approx \kappa_0^{1/3} \approx \exp\left[-\frac{1}{\alpha_{GUT}}\right] \tag{11.4}$$

11.0.5 Grand Unified Theories

The physicists who constructed the Grand Unified Theories (GUT) started from an experimental fact : the three couplings of all interactions except gravity begin to converge at a energy scale M_X. (See the book Ross, 1984, [14] and all references therein). These theories are known to unify the three interactions (except gravity), but to possess two weaknesses : first they leave too arbitrary parameters and no interaction with gravity. On the contrary, formula (10.10) proves that gravity is nothing else than some yet to be determined nonperturbative quantum effect (instanton, tunnelling, of ELSE!) of these Grand Unified Theories, so these GUT entirely contain gravity. Furthermore, (10.10) proves that, via this non perturbative correspondence (we mean the logarithm or inversely the exponential in (10.10), the three couplings converge also to the gravitational coupling (the mass of the electron, or more generally the mass of the particles). It then appears that these theories contain the necessary information to compute the masses, and thus should at the end contain no arbitrary parameter. In other words, (10.10) seems to prove that, in fact, the Grand Unified Theories unify all four interactions.

11.0.6 The GUT energy scale

The value of the GUT energy scale is $M_X = 2 \times 10^{15} GeV$. The value for such a mass, using our principle : "the simplest formula gives the mass", is :

$$M = \sqrt{\alpha_{em}}\kappa_0^{-1}(\Omega^{1/2} H) \tag{11.5}$$

We recall that at this scale, $\alpha_{em} = \alpha_{GUT}$, thus we find $M = 7.4 \times 10^{16} GeV \approx 37 M_X$.

12 Consequences for the nature of gravity

12.0.7 Classical spacetime

We simply add that we always placed ourselves in a classical spacetime. What has been thought to be a breakdown of spacetime at Planck length is in our theory simply the radiuses of particles. These radiuses cannot be smaller, because they are controlled by κ_0. Thus, we simply use a classical spacetime, and insert in Feynman integrals the physical cut-off determined by κ_0^{-1}. In this classical spacetime, there can be two cases. We see that formula (10.10) links the mass of the electron with what seems to be euclidian instantons. So there are two cases for gravity : it can be a purely quantum relic (first case) (nonperturbative correction) to the $SU(5)$ Yang-Mills theory (or $SO(10)$, which seems to take into account the masses of the neutrinos), or it can be a nonperturbative phenomenon which still needs to be quantized (second case).

12.0.8 The status of Einstein's theory (first case)

However, there still is a problem with Einstein's theory. We believe that each non abelian theory possesses its real, observable nonperturbative (only attractive) purely quantum effects (first case). In the case of $SU(5)$, these are gravity. So, what is now the status of Einstein's theory? Clearly, Einstein's theory is an effective theory which gives account for this purely quantum effects, when these effects are seen at the classical scale (in this case, there is no classical gravity, gravity is only a PURELY QUANTUM RELIC). Then, from the classical point of view, there is no gravity. However, if Einstein's gravity is just an effective theory to describe at the classical scale what we see of a purely quantum phenomena, THIS GRAVITY SHOULD NOT BE QUANTIZED BY THE USUAL FORMALISM AVAILABLE FOR THE OTHER INTERACTIONS. Simply because we cannot quantize something which is already a purely quantum correction. Indeed, we quantize a interaction to determine the deviations from the classical theory. In the hypothesis of emergent gravity, gravity is already one of these deviations (what we meant by : there is no classical gravity). Beyond emergent gravity, the gravity we have been led to a Poincare's gravity. In one of his works, Poincare emitted the hypothesis that gravity simply did not exist as an interaction, but was only a relic of electromagnetism. To explain this, he gave the example : if the proton had an electric charge slightly different from minus the charge of the electron, the planets would have a slight global excess of electric charge and electromagnetism could display, at their scale, a phenomenon called gravity, which had no existence by itself, but was only a relic of electromagnetism. In the example of Poincare, of course we obtain a repulsive gravity, and we do not know if he has been imagined, by this procedure, attractive gravities. Thus, formula (10.10) proves, in our opinion, that we have arrived at a Poincare's gravity : a (quantum) relic of the other interactions, which happens to have effects that can be seen at the classical level. To give an example, we consider the no-go theorem of Weinberg-Witten which asserts that "an interacting graviton cannot emerge from an ordinary quantum field theory in the same spacetime". With our Poincare-like gravity, we can stay in four dimensions, because we do not need to avoid Weinberg-Witten theorem : what our analysis makes us believe at this stage, is THAT THERE IS NO GRAVITON AT ALL. (we are aware that this possibility has already been considered as an eventual hypothesis, but we assert that formula (10.10) transforms it in THE SIMPLEST AND MOST PHYSICAL CONJECTURE, SUGGESTED BY EXPERIMENT itself, taking into account our present knowledge). What would mean anyway, a boson for inertia, whereas at the quantum level, we cannot define inertia for particles (because of their wavy nature, and because inertia can only be defined for classical bodies). In a future work, we will try to treat this problem and particularly to discuss the wave/particle duality problem, which is at the center of this question.

12.0.9 The status of Einstein's theory (second case)

If, and we now place ourselves in the second case, gravity is now a nonperturbative euclidian-instanton-like phenomenon coming from the $SU(5)$ Yang-Mills theory, we mean a phenomenon which has its classical and quantized regimes, THERE IS NO GRAVITON EITHER. Indeed, independently on how we can retrieve Einstein's theory from this instanton-like phenomenon, the quantization of the theory can be made by the rules of quantization of these supposed instantons and not by the rules of quantization of general relativity. So we arrive at the same conclusion : WE SHOULD NOT TRY TO QUANTIZE GENERAL RELATIVITY (which still is an effective theory). This idea is confirmed by our first hypothesis of the quantum equation of gravity : a tensorial equation can take into account quantum features of the theory. So even in the second case which we are now studying, it is possible that general relativity not only takes into account the classical behavior of our classical instanton phenomenon, but also a part of (if not all) the quantum side of this instanton phenomenon.

12.0.10 A first try

We now try to see how euclidian instanton effects can be put by hand in the usual equations, written in Minkowskian spacetime. Starting with the gauge theory

$$S = \int F_{\mu\nu} F^{\mu\nu} d^4x \tag{12.1}$$

we add to this lagrangian the topological :

$$S = \int F_{\mu\nu} F^{\mu\nu} d^4x + i\theta \int F_{\mu\nu} \tilde{F}^{\mu\nu} d^4x \tag{12.2}$$

We should obtain our Poincare's gravity, because in the path integral, $e^{iS/\hbar}$ will display the factor which is on the second hand of equation (10.10). We clearly are aware that this first solution, with no change, may lead to inconsistencies, but starting from it, we should be able to improve this first try.

12.0.11 Unification unifies the contingent and the fundamental

Unification unified the smallest and largest scales of the universe. However, it unifies now the most contingent and the most fundamental : κ_0, and gravity, appears to be generated by some relic which is a quantum correction of the three other forces : it is the most contingent, it is so contingent that it does not exist by itself (classically). And here is κ_0, the most fundamental constant in physics, which governs everything in the universe.

12.1 A few remarks

We see that up to constants of order unity, the exponents of κ_0 determine all quantities in the universe. The temperature of the cosmic background radiation is $T = 2.73K$ approximated by

$$T = (\kappa_0)^{-1/2}(\Omega^{1/2}H) = 10.09K \tag{12.3}$$

The total entropy of the universe, the total number of photons are governed by the exponent $(\kappa_0)^{-3/2}$, the ratio eta of the total number of baryons to the total number of photons is governed by the ratio $(\kappa_0)^{1/6}$, the value of θ in the strong CP problem could be $(\kappa_0)^{1/6}$, coming from a yet unknown procedure of unification, the total mass of the universe $\tilde{M}$ is governed by $(\kappa_0)^{-2}$.

13 The mass gap problem

The mass gap problem, presented by A. Jaffe and E. Witten (Jaffe, Witten, 2000, [15] as a millennium problem of the Clay Mathematics Institute, corresponds to the following question : is there, from the side of gauge theories, a mechanism that provides us with an energy gap : "there must be some constant $\Delta > 0$, such that every excitation of the vacuum has energy at least Δ". And they add : "Since the vacuum vector Ω is Poincare invariant, it is an eigenstate with zero energy, namely $\widehat{H}\Omega = 0$". The problem is to establish the existence of a non-trivial quantum Yang-Mills theory that exhibits a mass gap, the existence including definite axiomatic properties. Furthermore, the supremum of such Δ is the mass m, and it still has to be proved that $m < +\infty$. Of course the relations that we proved concerning the masses of the fundamental particles give an negative answer to this problem. Such a condition $m \geq \Delta > 0$, from the side of gauge theories, even in the case in which it is possible mathematically, would ruin the whole Yang-Mills method : it would be in contradiction with the fact that the masses tend to zero since there are proportional to some function of $\dot{a}$, multiplied by the factor $1/a$ which tends to zero, whereas $\dot{a}$ varies only very little compared to a. So, we turn to another problem which is : in the hypothesis $m = \tilde{m}H$, try to establish the mass gap for $\tilde{m}$, with the existence of a $\tilde{\Delta}$, defined by $\Delta = \tilde{\Delta}H$. However, this problem also has a simple solution, which is yes, and which is available in any case, that is for any Yang-Mills theory. Why this? Because any quantum Yang-Mills theory, as any quantum theory, verifies the relations of incertitude of Heisenberg, and we shall see that in this case, there is always a mass gap Δ. Indeed, in our context, equation (5.2) is equivalent to $\Delta = H$ and thus to $\tilde{\Delta} = 1$. We recall that the principle of the proof : we use Heisenberg's incertitude relations to find a mass gap depending on the interval of time of the observation, and we add that this interval of time is necessarily less that the age of the universe.

13.1 The necessarily breaking of Poincare invariance

The quantum equation of gravity does not satisfy anymore the property of general covariance, and is not even Poincare invariant. We explain now why this is a necessary condition on every theory of quantum gravity. In fact, Poincare invariance, and general covariance, will be lost once we renormalize the gravitational constant. Indeed, the equations of general relativity are

$$R_{ik} - \frac{1}{2}R = 8\pi G T_{ik} \tag{13.1}$$

If we stick to these equations, and try to guess to what changes would lead quantum corrections, we return directly to our ideas for the construction of the quantum equation of gravity. Using renormalization theory, we deduce that quantum corrections lead to the fact that the couplings run with energy. The equation is tensorial, but in any tensorial equation, the constants also have to be tensors : the constants are constant functions of space-time variables, that have to keep their value in any change of coordinates. If, after the first order corrections, G depends on energy, and since energy is not Poincare invariant, the Poincare invariance is automatically broken, as we saw in [3]. However, we also saw in [3] that the vacuum is obtained when the right hand side of a tensorial equation of gravity vanishes, in which case the gravitational coupling disappears, leading to the fact that all tensorial equations are equivalent. We used this property to deduce that quantum gravity and general relativity are equivalent in vacuum. Thus, in vacuum, the quantum equation of gravity is Poincare invariant again, and even generally covariant. Also, the vacuum of the equation of quantum gravity is zero. In this equation, Λ does not represent the energy of the vacuum, it is a term another term representing (exactly : supposed to mimic) the nonperturbative corrections to Einstein's theory. However, we have the relation $\Omega_\Lambda = \Omega_{TOT}/2$. If we note Ω_ϵ the part of Ω_{TOT} corresponding to real matter, we have $\Omega_{TOT} = \Omega_\epsilon + \Omega_\Lambda$ and thus : $\Omega_\Lambda = \Omega_\epsilon$. The vacuum is characterized by the relation $\Omega_\epsilon \to 0$, which gives straightforwardly $\Omega_\Lambda = 0$.

13.2 A negative answer to the mass gap problem

We now return to our problem of finding a mass gap Δ. The masses can be written :

$$m = \tilde{m} f(\dot{a}) \frac{1}{a} \tag{13.2}$$

with $\tilde{m}$ a dimensionless constant mass, and $f(\dot{a})$ an undetermined function of $\dot{a}$. However, the mass gap can be asked to m, in which case we can prove that the answer is no, or can be asked to $\tilde{m}$, in which case we can prove the answer is yes. We now ask the question to m. We place ourselves in the case $\theta(a) = \theta_0 a^2$. This case implies an equation for a which is (17.5) of [1] :

$$\dot{a} = \frac{1}{\sqrt{2\theta_0}} \sqrt{\ln(a/a_0)} \tag{13.3}$$

We know from (14.7) that in any case, there exists $C > 0$ and constant, such that :

$$m \leq C \frac{\dot{a}^4}{a} \sim \frac{\ln^2(a/a_0)}{a} \tag{13.4}$$

So nothing can prevent this mass from tending to 0 when $a \to +\infty$, and there can be no mechanism, coming from gauge theories, which keeps the mass :

$$m \geq \Delta > 0 \tag{13.5}$$

This mechanism would be in contradiction with the forever expansion of the universe. Indeed, the quantum equation of gravity implies $\ddot{a} > 0$, and the existence of a minimum of a noted a_0, for which $\dot{a}_0 = 0$. Such conditions on any function $a(t)$ imply that $a(t) \to +\infty$ when $t \to +\infty$. Indeed, $\dot{a}$ is strictly increasing and starting from zero, becomes strictly positive, such that all the tangent to the curve become strictly increasing. From this, being convex, $a(t) \to +\infty$, because it is greater than all its tangents.

13.3 A positive answer to the mass gap problem

If the question of the mass gap is asked to $\tilde{m}$, then the answer is yes and comes simply from the Heisenberg's incertitude relations. To distinguish any kind of positive energy from the value zero, this energy must have a strictly positive value, of course greater than the uncertainty coming from Heisenberg's incertitude relations. These last relations thus imply that the energy under consideration must take some value Δ such that $\Delta \times T \geq h$, and h is Planck's constant. Here, T is the value of the interval of time needed to observe the energy Δ. If we take for T the greatest interval of time possible, the age of the universe, we see that the Heisenberg relation implies, using the condition $\hbar = c = 1$, $\Delta \geq 1/t$, where now t is the age of the universe. Furthermore, we ask for this energy Δ to be written in the dimensionless units of $\tilde{m}$. We thus are studying $\tilde{\Delta}$, defined by :

$$\Delta = \tilde{\Delta} f(\dot{a}) \frac{1}{a} \tag{13.6}$$

In these dimensionless units, $\tilde{\Delta}$ is the smallest energy possible, strictly greater than zero. Here $\tilde{\Delta}$ is a function depending on the cosmological time t, and we want to know if it keeps values greater than some strictly positive given value, during all times. In other words, we want to prove that

$$\frac{1}{\tilde{\Delta}} \leq Cte \tag{13.7}$$

We proved the condition $\Delta \geq 1/t$, which implies that (15.7) is equivalent to :

$$\frac{1}{\tilde{\Delta}} = f(\dot{a}) \frac{1}{a\Delta} \leq f(\dot{a}) \frac{t}{a} \leq Cte \tag{13.8}$$

This condition can be proved, provided we have in our quantum equation the right function $\theta(a)$. From the mathematical point of view this is enough, because finding only one function $\theta(a)$ is equivalent to finding the mathematical mechanism, which was asked for, in the formulation of the problem. To solve the problem from a physical point of view, we have to take the particular function $\theta(a)$, which has been proved the most physical in [3] :

$$\theta(a) = \theta_0 a^2 \tag{13.9}$$

In this case, we can apply (12.3) :

$$\dot{a} = \frac{da}{dt} = \frac{1}{\sqrt{2\theta_0}} \sqrt{\ln(a/a_0)} \tag{13.10}$$

where a_0 is the minimum value for the cosmological parameter a. We saw earlier that there are two possibilities for the universe before it reached the value $a = a_0$: in one first physical case, a_0 is only a minimum and the universe has shrunk once to this value before expanding again, or in another theoretical case, the values of θ are quantized, and before the condition $a = a_0$, there has been a time of constant θ, with an exponential growth of a, with $a \to 0$ for $t \to -\infty$. However, from the time when $a = a_0$, the age of the universe has been

$$t = \sqrt{2\theta_0} \int_{a_0}^{a} \frac{dx}{\sqrt{\ln(x/a_0)}} \tag{13.11}$$

and we can take this time to be the longest interval of time for the observation of a particle. Indeed, in case the universe has grown to this value a_0 with an exponential law, a comes from zero, and there has been a time $t_1 < t_0$, where the radius where so near the value zero that the concept of particles as we know them could not exist. So before time t_1, the particle we study could not exist, and thus has not been observed. From t_1 to t_0, there is only a finite interval of time. Such a finite interval of time $(t_0 - t_1)$ counts for

$$f(\dot{a})\frac{t_0 - t_1}{a}$$

in (12.8), and it is easy to check that this term does not make arise any problem in the following proof. The most interesting and physical case is when a_0 is only a minimum of a. We proved in [3] that in this case, time before a_0, runs backwards and not forwards. Indeed, the big bang, that is to say the universe at the value a_0, does not possess a past from which the universe has shrunk to a_0 and a future, into which the universe is growing again. On the contrary this point has no past but two futures, probably equivalent. In other words, and this shall be proved carefully, if we could look through the big bang, we would not see some kind of past of the big bang, or cause of the big bang, but we would see one future, acting as a cause of ourselves, in a twin universe in which the effect is always before the cause. This is clearly the logical solution for the existence of the universe : there are twin universes, each of them being the cause of the existence of the other. In this case, the greatest interval of time that can be considered is the interval of time from a_0 until now. We make the change of variables $x = a_0 y$ in the integral (12.11), and our condition becomes :

$$f(\dot{a})\frac{t}{a} = \frac{f(\dot{a})}{a}\sqrt{2\theta_0 a_0^2}\int_1^{a/a_0}\frac{dy}{\sqrt{\ln y}} \leq Cte \tag{13.12}$$

There is no problem for the integral at the values $y \approx 1$, because :

$$\int_1^2 \frac{dy}{\sqrt{\ln y}} < +\infty$$

For the smallest values of a, the expression in (12.12) is less or equal than a constant, because a does not tend to zero but to a strictly positive value a_0. Furthermore, when $a \to a_0$, we have $\dot{a} \to 0$. The worst behavior which we consider for $f(\dot{a})$ is the case $m \sim \Omega^{1/2}H$, equivalent to $f(\dot{a}) = \sqrt{1+\dot{a}^2}$. When $\dot{a} \to 0$, we find $\Omega^{1/2}H \sim 1/a_0$, which is a constant, so $f(\dot{a})t/a$ has no problem in this limit. For $a \to +\infty$, we compute an equivalent of the integral. If X is a real variable, the derivative of $X/\sqrt{\ln X}$ is :

$$\left(\frac{X}{\sqrt{\ln X}}\right)' = \frac{1}{\sqrt{\ln X}} - \frac{1}{2(\ln X)^{3/2}} \sim \frac{1}{\sqrt{\ln X}} \tag{13.13}$$

Here the symbol $f(X) \sim g(X)$ means that the ratio $f(X)/g(X)$ tends to 1 when $X \to +\infty$. We integrate this relation, finding :

$$\int_1^X \frac{dy}{\sqrt{\ln y}} \sim \int_e^X \frac{dy}{\sqrt{\ln y}} \sim \int_e^X \left(\frac{y}{\sqrt{\ln y}}\right)' dy \sim \frac{X}{\sqrt{\ln X}} \tag{13.14}$$

We thus find that, for $a \to +\infty$:

$$\frac{f(\dot{a})t}{a} \sim \frac{f(\dot{a})\sqrt{2\theta_0}a_0}{a}\int_1^{a/a_0}\frac{dy}{\sqrt{\ln y}} \sim f(\dot{a})\frac{\sqrt{2\theta_0}}{\sqrt{\ln(a/a_0)}} \tag{13.15}$$

We also have

$$\dot{a} = \frac{1}{\sqrt{2\theta_0}}\sqrt{\ln(a/a_0)}$$

and we obtain :

$$\frac{f(\dot{a})t}{a} \sim \frac{f(\dot{a})}{\dot{a}} \tag{13.16}$$

We first make one important remark : equation (12.16) is the key equation to prove the mass gap problem, and is at the same time equivalent to the following relation :

$$\frac{t}{a} \sim \frac{1}{\dot{a}}$$

or in other words, (12.16) is equivalent to

$$t \sim \frac{1}{H} \tag{13.17}$$

Equation (12.17), equivalent to (12.16), itself equivalent to the solution of the mass gap problem, has been experimentally observed (Bennett and al. 2003, [5]). It is also really important to notice that the result (12.16) is independent of the value of θ_0. Indeed, physically, we would like to see integers emerging in the theory, because we preview that unification will be a theory involving only integers. To obtain these integers, we would like to find $\tilde{\Delta} = 1$, or at least find that the value of $\tilde{\Delta}$ does not involve any parameter of the equation, like θ_0. The condition solving the mass gap problem is then

$$\frac{f(\dot{a})}{\dot{a}} \leq Cte \tag{13.18}$$

Now, if we note a_1 and $\dot{a}_1$ the present values respectively of the radius of the universe and of its time derivative, the complete relation for $\dot{a}$ is :

$$\dot{a}^2 = \dot{a}_1^2 + \frac{1}{2\theta_0} \ln(a/a_1) \tag{13.19}$$

We saw that the value of $\dot{a}_1$ should be around unity, at most 20, the greatest value considered in [3] being 7.07. Now, if we just look at the behavior of $\dot{a}$, we see that when the universe will be $(\kappa_0)^{-1}$ times bigger that it is today, we recall that for this value, the radius of this future universe will be to the present radius what the present radius is today to the Planck length, for this value we said, and for $\dot{a}_1 \approx 20$ the value of $\dot{a}$ will only be around 23. Indeed, we proved in [2] that $2\theta_0 = 1$. We thus understand that the only possible values of $\dot{a}$ are around unity, and since $f(\dot{a})$ is a continuous function, (12.18) is satisfied since $\dot{a}$ stays around unity. We add that the value of $(\kappa_0)^{-1}$ has been considered as the greatest value ever imaginable in our world. This means that in fact the ratio a/a_1, for one yet unknown reason, has no chance to go beyond this value. We should imagine that to one point, when this ratio becomes too big, the laws of physics as we elaborated them should breakdown completely. We should probably have a universe with only particles with vanishing masses.

13.4 Another proof of the existence of a mass gap

The former proof is the most general : it uses no further hypothesis on $f(\dot{a})$. We can also prove the existence of the mass gap, that is to say (12.18), by turning to experiment to consider the most physical function $f(\dot{a})$, or even to the quantum cosmological model under consideration, plus find a physical principle that will permit us to compute $f(\dot{a})$. We saw earlier that we could find the best estimates for the masses of the fundamental particles when we supposed that these were proportional to

$$m \sim \Omega^{1/2} H = \frac{\sqrt{1+\dot{a}^2}}{a} \tag{13.20}$$

or eventually to

$$m \sim H = \frac{\dot{a}}{a} \tag{13.21}$$

For example, the computation of the mass of the electron is in perfect agreement with (12.20) but does not rule out completely (12.21). In these cases, we can compute $f(\dot{a})$: (12.20) leads to the relation :

$$f(\dot{a}) = \sqrt{1+\dot{a}^2} \tag{13.22}$$

and we have :

$$\frac{f(\dot{a})}{\dot{a}} = \frac{\sqrt{1+\dot{a}^2}}{\dot{a}} \tag{13.23}$$

This is a decreasing function of $\dot{a}$, and also a decreasing function of time since $\ddot{a} > 0$. So $f(\dot{a})/\dot{a}$ decreases to 1 as $t \to +\infty$ and we now find the exact relation for $\tilde{\Delta}$:

$$\tilde{\Delta} = 1 \tag{13.24}$$

Equation (12.21) is even simpler and in this case

$$\frac{f(\dot{a})}{\dot{a}} = 1 \tag{13.25}$$

and equation (12.24) is still satisfied. We can also turn to a physical principle to be associated with our quantum cosmological model. We can suppose that the total number of baryons is constant in the universe, which leads to the relation :

$$m \sim \frac{(1+\dot{a}^2)^2}{a} \tag{13.26}$$

which equation (14.7) of [3] p 209. This behavior of m is too bad, and (12.26) cannot improve the general proof of section 12.3. On the contrary, if we use our quantum cosmological model with the additional hypothesis of the constancy of the intensity of the gravitational interaction, we use equation (14.3) of [3] p 209 to deduce that Newton's constant is proportional to

$$G \sim \frac{\kappa_0}{\sqrt{\epsilon}} \sim \frac{1}{\Omega H^2} \tag{13.27}$$

The constancy of the gravitational charge is :

$$G^{1/2} m \sim Cte$$

From this equation, we deduce directly (12.20) and conclude.

13.5 A third proof of the mass gap problem

There is finally a special case of our class of quantum cosmological models for which no additional principle is necessary to compute $f(\dot{a})$, and directly conclude to the existence of a mass gap. This is the limit case $\theta \to +\infty$. Equation (15.18) of [1] reads :

$$\frac{\ddot{a}}{a} = \frac{1}{4\theta(a)}$$

In the case $\theta \to +\infty$, the last relation becomes $\ddot{a} = 0$, and thus $\dot{a} = \lambda$, where λ is a constant. We integrate this relation and find $a(t) = \lambda t$, since we still can decide to choose $t = 0$ for $a = 0$. We thus obtain :

$$f(\dot{a})\frac{t}{a} = \frac{f(\lambda)}{\lambda} = Cte \tag{13.28}$$

and (12.12) of the present article is satisfied, which proves once again the existence of a mass gap.

13.6 A simplified picture of the proof

We give a simplified proof, in the picture in which the masses are proportional to Hubble's constant :

$$m = \tilde{m} H \tag{13.29}$$

From the observations of Bennett and al. 2003 [5], or from our theoretical analysis in [1], section 17, we know that the age of the universe is :

$$t \approx \frac{1}{H} \tag{13.30}$$

The smallest energy Δ that can be distinguished from zero is :

$$\Delta \approx \frac{1}{t} \approx H \tag{13.31}$$

Using now :

$$\Delta = \tilde{\Delta} H \tag{13.32}$$

we find again the right formula :

$$\tilde{\Delta} \approx 1 \tag{13.33}$$

In fact, we see further that this condition implies that in the relation :

$$m = \tilde{m} H \tag{13.34}$$

the incertitude on the value of $\tilde{m}$ is around unity, so there is no value of this variable that can be distinguished from its nearest integer. This way, we can suppose that $\tilde{m}$ only takes integer values. This formulation in fact is equivalent to the condition $\tilde{\Delta} = 1$.

Email address : cristobal.real@hotmail.fr

References

[1] C. Réal, *Tensorial Quantum Gravity and the Cosmological Constant Problem*, arXiv:0711.1441.

[2] C. Réal, *Physical Unification and Masses*, in *Unification and the Masses of the Fundamental Particles*, Editions Cristobal, 2007.

[3] C. Réal, *Unification and the Masses of the Fundamental Particles*, Editions Cristobal, 2007.

[4] G. Lochak, *The Equation of a Light Leptonic Magnetic Monopole and its Experimental Aspects* Z. Naturforsch. 62a, 231-246 (2007).

[5] C. L. Bennett and al., *First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations : Preliminary Maps and Basics Results*, astro-ph/0302207, Astrophysical Journal.

[6] L. De Broglie, *La Thermodynamique de la particule isole*, Gauthier-Villars, 1963 .

[7] P. J. E. Peebles, *Principles of Physical Cosmology* Princeton : Princeton University Press, 1993.

[8] L. Landau, E. Lifchitz, *Physique Theorique Electrodynamique Quantique; Quantum Electrodynamics* Ed. Librairie du Globe; Editions Mir.

[9] Yao and al., J.Phys. G33, 1, 2006.

[10] F. Mandl, G. Shaw, *Quantum Field Theory*, John Wiley and Sons, 1984, reprinted in 1995.

[11] A. Pich, *The Standard Model of Electroweak Interactions* in *The Standard Model and Beyond*, Editions Frontieres, 1995, p. 1-41.

[12] M. Consoli, F. Jegerlehner and W. Hollik, *Z Physics at LEP* CERN Yellow Report CERN 89-08, Vol. 1, CERN Geneva, 1989, p.55.

[13] K. Yagi, T. Hatsuda, Y. Miake, *Quark-Gluon Plasma*, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, 2005.

[14] G. G. Ross, *Grand Unified Theories*, Frontiers in Physics, 1984.

[15] A. Jaffe, E. Witten, *Quantum Yang-Mills Theory*, Millennium Prize Problem, Clay Mathematics Institute.